

GROUND STATE OF THE DOUBLE-WELL CONDENSATES INTERACTING WITH TRAP OSCILLATIONS

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In the present paper it is shown that the interaction between classical anharmonic oscillations of a double-well trapped condensate and excited Josephson states corresponding to a large enough initial disbalance of the particle number generates their bound state. The bound state can realize an absolute minimum of the thermodynamic energy. The transition to new ground state is a second-order phase transition. The existence of the bound state implies that the Josephson states can be detected by observing the change in the condensate shape.

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1. INTRODUCTION

The realization of two-component condensates [1] as well as condensates in a double-well potential [2] has attracted considerable interest, both experimental and theoretical (see, e.g., [3] and references therein) to the phenomena caused by phase coherence of two condensate modes. In the work [2] spatial quantum coherence was observed by means of interference pattern in two overlapping condensates. This interference pattern was conformed in the work [4] with the numerical simulation of the Gross-Pitaevskii equation. In the work [5] coherent oscillations of the relative populations have been observed in driven two-component condensates with different internal states. As is well known, a bright manifestation of phase coherence is Josephson effect. In numerous studies devoted to the Josephson effect in systems of two condensates in different internal states [5], [6] or in a double-well potential [7]-[8] coherent Josephson oscillations are considered for various dynamical regimes caused by the competition between tunneling and intra-condensate interaction (nonlinearity). In the work [7] the Josephson coupling energy is calculated for small-amplitude oscillations in a double-well potential. There are estimated damping effects due to the normal currents at finite temperature. In the work [8] it is shown that for a relatively weak interaction, the particle number oscillations between the condensates are complete. They are suppressed when the total number of atoms in the condensates exceeds a critical value and the behavior of the system is governed by the nonlinearity. Nonlinear Josephson-type oscillations in the relative oscillations of driven two-component condensates are studied in the work [6]. Decoherence effects and quantum corrections to mean-field solutions have been considered in [9], [10]. In [11] the damping effects (even at zero temperature) of the Josephson current are derived by means of an functional integral approach. The detailed treatment of the nonlinear classical dynamic of the condensates in a double-well

potential has been performed in the works [12],[13]. In the work [14] the quantum and thermal fluctuations of the phase are studied for the condensates in the double well potential.

It should be emphasized that experimental observation of the Josephson effect is difficult because the small energy splitting associated with Josephson coupling means that thermal and quantum fluctuations will destroy the phase coherence between two condensates even at the lowest achievable temperatures [11], [14]. While the energy splitting can be increased, for instance, by lowering of the barrier height, it then becomes comparable with that of motion states of the condensates.

However, the problem of the interaction between the Josephson degrees of freedom and states of motion (oscillations) of the trapped condensate has yet to be analyzed. The present paper focuses on a mechanism of formation of the bound state of the Josephson degrees of freedom and trap oscillations due to their interaction. The mechanism proposed in what follows may be important for detection of the excited Josephson states.

The results achieved in this paper may be listed as follows.

1. To formulate the problem of the interaction between the Josephson and oscillation degrees of freedom in an adequate manner, the quantized spectrum of the particle number generated by Josephson coupling is derived. The states of this spectrum represent the quantum analogue of the nonlinear coherent Josephson oscillations considered in [12]. In what follows, the states of the quantized spectrum are named as Josephson states. The spectrum is highly nonequidistant and has the logarithmic singularity in the density of states at the $2E_J$ energy, where E_J is the Josephson coupling energy. As is shown in what follows, any Josephson state can be realized by means of a given initial disbalance of the particle number in two condensates.

2. There is considered the interaction between the trap oscillations and excited Josephson states corresponding to the large enough initial disbalance of the particle number. It is shown that this interaction is responsible for the formation of the bound state of $\bar{n}_m \gg 1$ oscillation quanta with the Josephson state corresponding to the

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definite initial disbalance. In the Thomas-Fermi approximation at $\mu \gg \omega_0$, where μ is the chemical potential of the condensates, ω_0 is the characteristic frequency of a trap, $\hbar = 1$, the bound state arises in the region of the dense oscillation spectrum. In this region the level separations are small in comparison with the harmonic oscillation frequency ω_0 . The equilibrium values of the oscillation quanta and initial disbalance of the particle number are coupled self-consistently and can realize an absolute minimum of the thermodynamic energy at the large enough interaction. The thermodynamic average $\bar{n}_m \neq 0$ generates the equilibrium distortion of the condensate shape. It allows us to detect the Josephson states by observing the change in the condensate shape.

2. THE QUANTUM SPECTRUM OF THE PARTICLE NUMBER

A Josephson coupling is realized for the condensates in an symmetrical/asymmetrical double-well potential formed by two different traps with a barrier between them [2]. The barrier is created by the laser light, and its height is directly proportional to the laser power and, thus, can be varied with ease. The proposed mechanism is also suitable for the condensates in different internal states in the same trap. Experimentally, this may be a superposition of two Rb⁸⁷ condensates in the $|F = 1, m_F = -1\rangle, |F = 2, m_F = 1\rangle$ states [5], [6]. In the case of a weak Josephson coupling, the basis states are the self-consistent ground states in two condensates separately. The wave function of the condensate with Josephson coupling takes the form of a superposition of these states, namely, $\Psi(\mathbf{r}, t) = \psi_1(\mathbf{r})a_1(t) + \psi_2(\mathbf{r})a_2(t)$, where $\psi_i(\mathbf{r})$ are the normalized solutions of the Gross-Pitaevskii equation, $a_i(t) = N_i^{1/2}(t)e^{i\theta_i(t)}$, $i = 1, 2$, $N_{1,2}$, $\theta_i(t)$ are the particle numbers and phases of each condensate.

As is well known [3], [7], [8], [13] the Hamiltonian of two condensates with Josephson coupling has the form

$$H_J - E_0 = E_C(\Delta N)^2 - 2E_J \cos \phi, \quad (1)$$

here $\Delta N = N_1 - N_2$, $E_C = \partial\mu/\partial N$, $\mu \equiv \mu_1 = \mu_2$ are the chemical potentials, N are the total particle numbers, $\phi = \theta_1 - \theta_2$ is the relative phase of the condensates. The quantities E_C , E_J depend on the total particle number N . (ΔN) , ϕ are canonically conjugate variables. In (1) the energy origin is the mean-field summary energy of the condensates, namely, $E_0 \equiv \mu N$.

The quantization of the Hamiltonian (1) produces the spectrum of the particle number in the Josephson potential $E_J \cos \phi$. As is shown in what follows, any Josephson state can be realized by means of a given initial disbalance of the particle number. For this reason, it is interesting to obtain the complete spectrum generated by the Hamiltonian of (1), and to represent it as a function of the initial disbalance.

The Schrodinger equation for the Hamiltonian (1) is derived by means of the quantization rule: $(\Delta N) \rightarrow -i\partial/\partial\phi$. As a result, we obtain Mathieu's equation

$$\left[-\lambda \frac{d^2}{d\phi^2} - 2E_J \cos \phi_r \right] \Psi = \varepsilon \Psi, \quad \varepsilon \equiv H_J - E_0. \quad (2)$$

For $\varepsilon > 2E_J$ this equation has a continuous spectrum. The states of this spectrum correspond to the classical states with unlimited phase change: $-\infty < \phi < +\infty$ that are named as the self-trapping states in the works [12, 13]. In the region $-2E_J < \varepsilon < 2E_J$ Eq.(2) has the discrete spectrum. It corresponds to the region of finite motion of Hamiltonian (1), where the relative phase changes within $-\arccos(\varepsilon/2E_J) < \phi < \arccos(\varepsilon/2E_J)$ for every ε . In the Josephson regime at $E_J \gg E_C$ [3], the number of levels in a well is large and the discrete spectrum is specified by the Bohr-Sommerfeld formula:

$$\begin{aligned} \nu(\varepsilon_\nu) &= \oint \frac{d\phi_r}{\pi} \Delta N(\phi_r; \varepsilon_\nu) \\ &= \oint \frac{d\phi_r}{\pi} \left[\frac{1}{E_C} (\varepsilon_\nu + 2E_J \cos \phi) \right]^{1/2} \\ &= \nu_c \left[E(\kappa) - (1 - \kappa^2)K(\kappa) \right]; \end{aligned} \quad (3)$$

$$\nu_c = \frac{8}{\pi} \left(\frac{E_J}{E_C} \right)^{1/2}; \quad \kappa^2 = \frac{\varepsilon_\nu + 2E_J}{4E_J}; \quad \nu_c \gg 1;$$

$K(\kappa)$ and $E(\kappa)$ are complete elliptic integrals of the first and second kinds. The density of states $\rho_d(\varepsilon_\nu)$ results from Eq.(5) and is equal to

$$\rho_d(\varepsilon_\nu) = \frac{1}{2\pi} \frac{d\nu}{d\varepsilon_\nu} = \frac{K(\kappa)}{\pi^2 \omega_m}. \quad (4)$$

The level separations in (3) are given by the following expression

$$\omega(\varepsilon_\nu) = \frac{d\varepsilon}{d\nu} = \frac{\pi\omega_m}{2K(\kappa)}, \quad \omega_m = 2(E_C E_J)^{1/2}. \quad (5)$$

At $(E_J/E_C)^{1/2} \gg 1$ we have the relation $\omega_m \ll 2E_J$. The quantity ω_m determines the maximum splitting of the levels in the Josephson well. In what follows, the states with $\nu < \nu_c$ are denoted as "libration" states.

In the region of $\nu \geq \nu_c$ the $\nu(\varepsilon)$ dependence and density of states are determined by the following expressions

$$\nu(\varepsilon) = \frac{4}{\pi^2} \left(\frac{E_J}{E_C} \right)^{1/2} \kappa E(\kappa^{-1}); \quad \rho_c(\varepsilon) = \frac{1}{2\pi^2} \cdot \frac{\kappa^{-1} K(\kappa^{-1})}{\omega_m}. \quad (6)$$

The states (6) with $\nu > \nu_c$ are named as the self-trapping states.

Eqs.(3), (6) imply that

$$\begin{aligned} \varepsilon(\nu) &\approx -2E_J + \omega_m \nu; \quad 1 \ll \nu \ll \nu_c, \\ \varepsilon(\nu) &\approx 4\pi^2 E_C \nu^2; \quad \nu \gg \nu_c. \end{aligned} \quad (7)$$

At the same time, it is easy to show that $d^2\varepsilon(\nu)/d\nu^2 < 0$ at $\nu < \nu_c$ and $d^2\varepsilon(\nu)/d\nu^2 > 0$ at $\nu > \nu_c$. At $\nu = \nu_c$ the curve $\varepsilon(\nu)$ has a flex point.

Since the energy is conserved, the state with a given ν value can be realized by means of the definition of the initial values of $(\Delta N)_0$ and $\phi(0)$. Namely, $\varepsilon(\nu) = E_C(\Delta N)_0^2 - 2E_J \cos \phi(0)$. Supposing that $\phi(0) = 0$, we obtain the following relation between ν and $(\Delta N)_0$

$$\varepsilon(\nu) = -2E_J + E_C(\Delta N)_0^2 \quad (8)$$

Using Eqs.(7), (8), we arrive at the expressions

$$\begin{aligned} \nu &= \frac{1}{2\pi} |(\Delta N)_0|; \quad \text{for the self-trapping (sf) states,} \\ \nu &= \left(\frac{E_C}{\omega_m}\right) (\Delta N)_0^2; \quad \text{for the "libration" (l) states.} \end{aligned} \quad (9)$$

Combining (4), (6), we come to the dependence

$$\rho_{d,c}(\varepsilon) \sim \omega_m^{-1} \ln |1 - \varepsilon/2E_J|^{-1}; \quad \varepsilon \rightarrow 2E_J^\pm \quad (10)$$

Thus, there appears a new logarithmic singularity at the boundary separating the libration and self-trapping spectra.

3. THE INTERACTION OF THE JOSEPHSON STATES AND CONDENSATE OSCILLATIONS

In this and next sections it is shown that the spectrum of the system can change drastically due to the interaction between the excited Josephson states (7)-(9) with large enough ν values and oscillations of the condensate.

The interaction can be realized by the following mechanisms. First, the interaction is secured if we allow for the dependence of E_C in Eq.(8) on the atom displacements. The latter are generated by the condensate oscillation. Second, the interaction can be realized by applying two-photon traveling-wave laser pulse with the Rabi frequency Ω . The pulse both creates the condensates with the different particle numbers and induces the interaction of atom's displacements with the excited Josephson states corresponding to the particle number disbalance created by the pulse. General description proposed in what follows is independent on the specific mechanism producing the interaction.

Let us consider the classical states of motion of the condensate. These states may be described in terms of the complex amplitudes $a^*, a = n^{1/2} e^{\pm i\varphi_1}$, where $n = \langle a|\hat{a}^+\hat{a}|a \rangle = |a|^2$ is the average number of quanta in the coherent state $|a \rangle$. The variables n, φ_1 are canonical. By classical state of motion we mean that it's number of quanta is very large, $n \gg 1$. It is convenient to specify the relation between the a, a^* amplitudes and \hat{a}, \hat{a}^+ operators by the following way: $a = N^{-1/2} \hat{a}$. At this, the commutator of a, a^* is equal to zero with macroscopic accuracy: $[a, a^*] = 1/N \rightarrow 0$. The Hamiltonian of the motion states can be written in the form: $N\varepsilon(n)$.

For the quasiclassical Josephson states with $\nu \gg 1$, the c_ν, c_ν^* amplitudes may be written in the form $c_\nu = \nu^{1/2} e^{i\varphi_2}$. However, it is convenient to rewrite $\varepsilon(\nu)$, c_ν in terms of the variable $x = |(\Delta N)_0|/N^{1/2} \gg 1$. Combining this inequality with the requirement that $x = |(\Delta N)_0|/N \ll 1$ we arrive at the conditions for the x values:

$$1 \ll x \ll N^{1/2}. \quad (11)$$

Using Eqs.(7),(8), we find that $\nu = \nu(x)$, $\varepsilon(\nu) = N(-E_J/N + E_C x^2)$. In general case, the $\nu(x)$ dependence is implicit. It is determined by Eqs.(3), (6), (8). However, in the particular cases of the "libration" ($\varepsilon(\nu) \ll E_J$) and self-trapping ($\varepsilon(\nu) \gg E_J$) states, the relations between ν and $(\Delta N)_0$ can be represented in a simple form, as is seen from (9). By means of Eqs.(8), (9), we come to the following expressions

$$\begin{aligned} H_0 &\equiv N\varepsilon_0(n, x) = N\left[-\frac{E_J}{N} + \varepsilon(n) + E_C x^2\right] \\ &= N\left[-\frac{E_J}{N} + \varepsilon(n) + \varepsilon_J(x)\right], \end{aligned} \quad (12)$$

$$\begin{aligned} c_\nu^{(sf)} &= N^{1/4} x^{1/2} e^{i\varphi_2} \equiv N^{1/4} c_x^{(sf)}; \\ c_\nu^{(l)} &= N \left(\frac{E_C}{\omega_m}\right)^{1/2} x e^{i\varphi_2} \equiv N \left(\frac{E_C}{\omega_m}\right)^{1/2} c_x^{(l)}. \end{aligned} \quad (13)$$

For any mechanism producing the interaction between two subsystems, it can be written in the form of a multiple Fourier series in φ_1, φ_2

$$\begin{aligned} H_{int} &= N \sum_{k_1 k_2} [g_{k_1 k_2}(N) a^{*k_1} c_\nu^{k_2} + c.c.] \\ &= N \sum_{k_1, k_2} [g_{k_1 k_2}^{(sf, l)}(N) n^{k_1/2} x^{\alpha k_2} e^{i(k_1 \varphi_1 - k_2 \varphi_2)} + c.c.]. \end{aligned} \quad (14)$$

Here $\alpha_{sf} = 1/2, \alpha_l = 1$, k_1, k_2 are integer. For the sake of simplicity, we disregard the phase-independent interaction. Using Eq.(13), we obtain

$$g_{k_1 k_2}^{(sf)}(N) = g N^{-1+k_2/4}; \quad g_{k_1 k_2}^{(l)}(N) = g N^{-1+k_2} \left(\frac{E_C}{\omega_m}\right)^{k_2/2}. \quad (15)$$

The constant g is specified by the concrete mechanism producing the interaction. Let us assume that the term with the phase $\phi_k^r = k_{1r} \varphi_1 - k_{2r} \varphi_2$, which varies anomalously slowly with time, can be set off in sum (14). It is possible to make under two conditions

$$\begin{aligned} k_{1r} \left(\frac{d\varepsilon(n)}{dn}\right) &= k_{2r} \left(\frac{d\varepsilon_J(x)}{dx}\right), \quad \varepsilon'_n \equiv \left(\frac{d\varepsilon(n)}{dn}\right), \\ x_m &= \frac{k_{1r} \varepsilon'_n}{2k_{2r} E_C}. \end{aligned} \quad (16)$$

$$\left(\frac{d^2 H_0}{dx^2}\right)_{x=x_m} (\Delta x)_{max} \gg \left(\frac{\partial H_{int}}{\partial x}\right)_{x=x_m}. \quad (17)$$

Eq.(17) is written with taking into account that H_0 , H_{int} are functions of a single dynamic variable, e.g., x . The ϵ'_n quantity defines the level separations of an oscillation spectrum.

As is shown in what follows, the condition (16) is equivalent to that of the minimum of the function $\varepsilon_0(n, x)$ over x . In turn, when the minimum exists, it can provide the principal contribution into the thermodynamic functions.

In addition, the condition (16) implies that the phase ϕ_k^r is an approximate integral of motion in the absence of changing x near x_m : $d\phi_k^r/dt \approx k_{1r}\partial H_0/\partial n - k_{2r}\partial H_0/\partial x \approx 0$.

The inequality (17) implies that the width of the near-minimum region is large at the characteristic interaction variation scale. From Eqs.(16), (17) one can obtain that time changing the ϕ_k^r phase is proportional to $(d^2 H_0/dx^2)_m \Delta x$, where Δx is the change in x near the x_m value. The maximum value $(\Delta x)_{max}$ specifies the width of the near-minimum region in that $d\phi_k^r/dt \sim \Delta x$. The estimation for $(\Delta x)_{max}$ is given in what follows. Thus, the leading term in sum (14) has the form

$$H_{int}^{(r)} = N g_k^{(sf,l)}(N; n, x) \cos \phi_k, \quad \phi_k = k_{1r}\varphi_1 - k_{2r}\varphi_2; \quad (18)$$

All remaining terms in this sum are rapidly oscillating perturbations and will be disregarded in this work. Here and below the index k in g_k and ϕ_k denotes a set of k_{1r}, k_{2r} . One can easily show that aside from the energy $H = N[\varepsilon_0(n, x) + g_k(N; n, x) \cos \phi_k]$ the system in question has an additional integral of motion: $n_0 = n/k_{1r} + x/k_{2r}$, $dn_0/dt = 0$. Owing to this, the condition (16) is equivalent to that of the minimum of $\varepsilon_0(n_0, x)$ over x at a given n_0 value, as is mentioned above. Using Eqs.(16)-(18), it is straightforward to write the Hamiltonian $H_m = H_0 + H_{int}^{(r)}$ near the minimum to the first nonvanishing order in Δx :

$$H_m = N \left[\varepsilon_0(n_m; x_m) + \left(\frac{d^2 \varepsilon_0}{dx^2} \right)_m (\Delta x)^2 - g_{km} \cos \phi_k \right], \quad (19)$$

$n_m = n_0 - k_{1r}x_m/k_{2r}$, $(d^2 \varepsilon_0/dx^2)_m = 2E_c$, $g_{km} = g_k^{(sf,l)}(N; n_m, x_m)$. Terms with the derivatives of H_{int} are absent in Eq.(19) due to the condition (17).

Using that $E_C \sim \omega_0(a/a_0)^{2/5} N^{-3/5}$ in the Thomas-Fermi approximation [15] (here a , a_0 are the scattering and oscillator lengths, respectively), we can represent the range of $1 \ll x_m \ll N^{1/2}$ in the form

$$\frac{1}{N} \left(\frac{Na}{a_0} \right)^{2/5} \ll \frac{k_{1r}}{k_{2r}} \left(\frac{\epsilon'_n}{\omega_0} \right) \ll \frac{1}{N^{1/2}} \left(\frac{Na}{a_0} \right)^{2/5}. \quad (20)$$

As is known [15], the relation $(Na/a_0) \gg 1$ occurs in the Thomas-Fermi approximation. However, $N^{-1/2}(Na/a_0)^{2/5} \ll 1$. Owing to this relation, the condition (20) (or, what is the same, the condition (16)) specifies the region of the dense oscillation spectrum, where $\epsilon'_n \ll \omega_0$. Both here and in what follows, we suppose that $k_{1r} = k_{2r} = 1$ for the sake of simplicity.

From Eq.(19) the $(\Delta x)_{max}$ can be estimated to be $(\Delta x)_{max} \sim (g_{km}/E_C)^{1/2}$. Hence, the condition (17) takes the form:

$$\frac{|x_m - n_m|}{x_m n_m} \ll \left(\frac{E_C}{g_{km}(N)} \right)^{1/2}, \quad (21)$$

In what follows (see Eq.(34)), it will be shown that the relation (21) is fulfilled with macroscopic accuracy.

4. THE GROUND STATE

At a fixed n_0 value the principal contribution to the partition function comes from the neighbourhood of the minimum at $x = x_m$. The expression for $Z(n_0; x_m; T)$ is equal to

$$\begin{aligned} Z(n_0; x_m; T) &= const \int_{-\infty}^{\infty} d(\Delta x) \int_{-\pi}^{\pi} d\phi_k e^{-\beta H_m(n_0, (\Delta x), \phi_k)} \\ &= \frac{const}{(\beta N E_C)^{1/2}} \exp[-\beta N \varepsilon_0(N; n_m; x_m) + \ln I_0(\beta N g_{km})], \end{aligned} \quad (22)$$

where $\beta = 1/T$, T is temperature, $I_0(x)$ is the modified Bessel function. Eq.(22) implies that the expression for the free energy of the system has the form

$$F = N \varepsilon_0(n_m; x_m) + \frac{1}{2} T \ln(\beta N E_c) - T \ln I_0(\beta N g_{km}), \quad (23)$$

Using Eq.(23), we come to the following equation for the \bar{n}_m value realizing the minimum of the free energy

$$\left(\frac{d\varepsilon_{0m}}{dn_m} \right)_{n_m=\bar{n}_m} = \left(\frac{dg_{km}}{dn_m} \right)_{n_m=\bar{n}_m} \cdot \frac{I_1(\beta N g_{km})}{I_0(\beta N g_{km})}, \quad (24)$$

here $I_1(x) = I'_0(x)$. In addition to \bar{n}_m , the thermodynamic average of $\cos \phi_k$ may be determined from Eqs.(22) or (23). This average is equal to

$$\langle \cos \phi_k \rangle_T = - \frac{\partial \ln Z}{\partial (\beta N g_{km})} = \frac{\partial F}{T \partial (\beta N g_{km})} = \frac{I_1(\beta N g_{km})}{I_0(\beta N g_{km})}, \quad (25)$$

The order parameters \bar{n}_m , $\langle \cos \phi_k \rangle_T$ describe new coherent state. There is the bound state of the \bar{n}_m oscillation quanta and Josephson state generated by the initial disbalance of the particle number that corresponds to the x_m value. In addition, this state has the equilibrium phase coherence factor $\langle \cos \phi_k \rangle_T$. The $\bar{n}_m \neq 0$ value provides the equilibrium distortion of the condensate shape. The aboveobtained equations imply that the shape distortion is self-consistently coupled to the x_m value defining the equilibrium initial disbalance of the particle number.

At $T = 0$ the \bar{n}_m value realizes the minimum of the thermodynamic energy

$$E_m = N[\varepsilon_0(N; n_m, x_m) - g_{km}(N; n_m, x_m)]. \quad (26)$$

To determine $\bar{n}_m(T=0)$, it is suitable to use the following consideration. As is well known, the level separations ϵ'_n change slowly in dependence on n within the range of dense (quasiclassical) spectrum. For this reason, we can suppose that $\epsilon'_n \approx \text{const} \equiv \omega_b \ll \omega_0$. On this assumption, the x_m value does not depend on n_m and the \bar{n}_m quantity is equal to

$$\bar{n}_m^{1/2} = \frac{g^{(sf,l)}(N)x_m^\alpha}{2\omega_b}, \quad \alpha_{sf,l} = \frac{1}{2}, 1, \quad (27)$$

(It is worth noting that $(\partial^2 E_m / \partial n_m^2) > 0$). Both here and in what follows, the denotation $g_{k=1}^{(sf,l)}(N) \equiv g^{(sf,l)}(N)$ is used. Taking Eq.(27) into account, one readily gets

$$\bar{g}_{1m}^{(sf)} = \frac{g^{(sf)2}(N)x_m}{2\omega_b}, \quad \bar{g}_{1m}^{(l)} = \frac{g^{(l)2}(N)x_m^2}{2\omega_b},$$

$$\begin{aligned} \frac{E_m^{(sf)}}{N} &= -\frac{E_J}{N} - \frac{g^{(sf)2}(N)}{8E_C} \left(1 - \frac{8E_C^2 x_m^2}{g^{(sf)2}(N)}\right), \\ \frac{E_m^{(l)}}{N} &= -\frac{E_J}{N} - \frac{g^{(l)2}(N)x_m}{16E_C} \left(1 - \frac{16E_C^2 x_m}{g^{(l)2}(N)}\right). \end{aligned} \quad (28)$$

The expressions for energies imply, first, that we obtain the minimum in the region of the dense enough oscillation spectrum, which satisfies the condition (20). The minimum corresponds to the formation of the bound state for the \bar{n}_m, x_m values. Second, as is seen from Eq.(28), the absolute minimum of E_m can be realized within the ranges of

$$1 \ll x_m < \frac{g^{(sf)}(N)}{8E_C}; \quad 1 \ll x_m < \frac{g^{(l)2}(N)}{16E_C^2}. \quad (29)$$

These conditions are met when the interaction matrix elements $g^{(sf,l)}(N)$ are large enough. Let us estimate the condensate parameters that are required for existence of the absolute minimum. In the Thomas-Fermi approximation, the inequality $g^{(sf)}(N) \gg E_C$ occurs provided the total particle number is not very large, namely,

$$N \ll \frac{g}{\omega_0} \left(\frac{a_0}{a}\right)^{8/3}. \quad (30)$$

In turn, the relation $g^{(l)2}(N) \gg E_C^2$ holds true within the range of

$$N^{0.1} \frac{g^2}{\Omega^{1/2} \omega_0^{3/2}} \left(\frac{a_0}{a}\right)^{0.6} \gg 1, \quad (31)$$

Here we use that $E_J = \Omega N$. The condition (31) is fulfilled for all admissible parameters, if $g^2 / (\Omega^{1/2} \omega_0^{3/2}) \sim 1$. It should be emphasized that the right-side inequalities in Eq.(29) are much stronger than the condition $x_m \ll N^{1/2}$.

The transition to the state with $\bar{n}_m \neq 0$, $\langle \cos \phi_k \rangle_T \neq 0$ is second-order. By imposing that $\bar{n}_m \rightarrow 0$ at the transition temperature, one gets from Eq.(24).

$$T_c^{(sf,l)} = \frac{g^{(sf,l)2}(N)x_m^\alpha}{\omega_b} \cdot N. \quad (32)$$

The dependences of the transition temperatures (32) on the total particle number are given by the expressions

$$T_c^{(sf)} \sim N^{-0.2}, \quad T_c^{(l)} \sim N^{0.8}. \quad (33)$$

Thus, the transition temperature $T_c^{(sf)}$ has the macroscopic smallness in comparison with the $T_c^{(l)}$ temperature. Along with the conditions (30), (31), this fact implies that the libration Josephson state forms the bound state with the condensate oscillation rather than the self-trapping state.

5. CONCLUDING REMARKS

We have found that the interaction between the Josephson and oscillation states results in new coherent ground state of the double-well trapped condensate. There is the bound state of the anharmonic condensate oscillation and excited Josephson state. The latter is specified by the definite initial disbalance of the particle number. The x_m value defining the disbalance is self-consistently coupled to the equilibrium number of the oscillation quanta entering the bound state. Along with the \bar{n}_m order parameter, this state has the equilibrium phase coherence factor $\langle \cos \phi_k \rangle_T$, where ϕ_k is the relative phase of the Josephson and oscillation modes.

It should be emphasized that the bound state arises in the neighbourhood of the minimum of $\varepsilon_0(n_0; x)$ over x , where the conditions (16), (17) are met. The condition (16) specifies the type of the interaction between the Josephson and oscillation degrees of freedom. In addition, it imposes the definite restrictions on the spectrum of the oscillation states which may effectively interact with the Josephson degrees of freedom. Namely, the density of oscillation states should be large enough in order to satisfy the conditions (20). For instance, in an asymmetrical double-well potential the energy $\epsilon(n)$ of its classical oscillation states has three branches. Two branches have energies of $\epsilon_{1,2}(n) \leq V_b$, here V_b is the barrier height. There are the maximums at $\epsilon_{1,2}(n_{max}) = V_b$. The third branch has both the energy $\epsilon_3(n) \geq V_b$ and the minimum at $\epsilon_3(n_{min}) = V_b$. Thus, the regions of the dense spectrum exist in the neighbourhood of the extremums of the functions $\epsilon_i(n)$ ($i = 1, 2, 3$). In the other words, in the neighbourhood of barrier top.

As is shown in section III, the condition (17) can be represent in the form (21). Substituting the aboveobtained expressions for \bar{n}_m, \bar{g}_{1m} into (21), we find that it takes the form

$$|x_m - n_m|^{(sf,l)} \ll \left(\frac{g_{1m}^{(sf,l)}(N)}{E_C} \right)^{1/2}. \quad (34)$$

This inequality is satisfied with macroscopic accuracy.

It is worth noting that the conditions (30), (31) of the absolute minimum of the thermodynamic energy are experimentally controlled by means of either the interaction matrix element g or the particle number N .

The existence of the bound state generates the equilibrium distortion of the condensate shape specified by the \bar{n}_m value. This mechanism can provide the experimental detection of the excited Josephson states. The latter can be observed by changing the condensate shape. In addition, the phase transition to the new ground state

occurs at $T = T_c$, where transition temperatures $T_c^{(sf,l)}$ are defined by Eq.(32).

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